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THE PREDICTION OF THE TRANSPORT PROPERTIES OF A TURBULENT FLUID

J. R. TYLDESLEY and R. S. SILVER
Department of Mechanical Engineering, The University, Glasgow, W.2, Scotland

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Abstract—The transport properties of a turbulent fluid are investigated using a simple model to represent the detailed fluid behaviour. In the model this is attributed to the motions of fluid entities of varying size, shape and velocity, and an analysis is made to find the effect on the whole system, of interactions and transport between the individual entities. The analysis enables expressions to be found for the diffusivities of mass, momentum and energy in terms of properties of the turbulence. Experimental values of the various diffusivity ratios compare favourably with the theoretical predictions. It is concluded that the model is a useful concept for predicting the bulk behaviour in turbulent flow.

NOMENCLATURE

a, b, c, semi-principal axes;
c_p, specific heat at constant pressure;
C, mass concentration in entity;
C_f, mass concentration in fluid;
D, molecular mass diffusion coefficient;
E, turbulent energy density;
$\bar{F}$, drag force on entity;
G, numerical constant;
K, molecular thermal diffusion coefficient;
$\dot{m}$, rate of mass transfer;
N_R, initial entity Reynolds number, $\frac{\rho R V_0}{\mu}$;
P, molecular Prandtl number;
P_T, turbulent Prandtl number;
$\dot{q}$, rate of heat transfer;
q, turbulent energy flux;
q_H, thermal energy flux;
Q, expected value of q;
Q_H, expected value of q_H;
r, R, entity scale parameters;
Re, flow Reynolds number;
R_y, correlation coefficient;
S, molecular Schmidt number;
S_T, turbulent Schmidt number;
t, time;
t_0, reference time;
T, temperature of entity;
T_0, initial temperature of entity;
T_f, mean temperature of fluid;
(u, v, w), fluid velocity;
(u', v', w'), mean fluid velocity;
u_0, u_y, fluctuating components of velocity;
(U, V, W), absolute entity velocity;
(U_r, V_r, W_r), relative entity velocity;
(U_0, V_0, W_0), initial relative entity velocity;
(X, Y, Z), coordinate axes.

Greek symbols

α, $= \frac{3K}{\rho c_p R^2 \psi'}$;
β, $= \frac{9\mu}{2\rho R^2 \psi}$;
γ, γ^+, numerical constants;
δ, turbulence microscale;
ε, diffusion coefficient due to entity migration;
ε_e, coefficient for turbulent energy diffusion;
$\varepsilon_H, \varepsilon_H^+$, coefficient for thermal energy diffusion;
$\varepsilon_m, \varepsilon_m^+$, coefficient for mass diffusion;

$\varepsilon_\mu, \varepsilon_\mu^+$, coefficient for momentum diffusion;
λ, distance travelled by entity;
λ^*, maximum value of λ,
$$= \frac{2\rho R^2 V_0 \psi}{9\mu};$$
μ, molecular viscosity;
ψ, ψ', ψ^+, shape distortion factors;
σ, turbulent shear stress;
τ, expected value of σ.

Superscript
$+$, used when transport between large entities is dominated by small entity migration.

1. INTRODUCTION

OVER the years many conceptual models have been proposed to account for the bulk behaviour of a turbulent fluid. In general these models have proved to be of great value in engineering, not necessarily on account of their accuracy in describing the microscopic details of a turbulent fluid, but because they enable certain bulk phenomena to be discussed in terms of a single scalar parameter whose value can be found by experiment and used for further predictions. The Prandtl mixing length and Reynolds flux concepts are typical examples of such models and their utility in engineering design is well known. Despite this, neither model is in itself capable of predicting any of the bulk properties of the fluid without recourse to experimental results.

In the flow of a perfect gas the bulk behaviour is again the result of a large number of individual fluctuating motions. In this instance the development of statistical thermodynamics provided a framework within which predictions of the cumulative effect of molecular motion could be made from knowledge of individual molecules. That these predictions proved to be accurate is essentially due to the faithfulness with which the molecular model represented the microscopic details of the substance. In some of the early work in this field the molecular model was crude in the light of present experience, but nevertheless the predictions were useful approximations.

In fluid turbulence there exist more complex phenomena than molecular motion in a perfect gas. The usual methods of statistical thermodynamics are difficult to employ because there exists no permanent entity comparable with the molecule, but if a suitable model could be devised there is no reason in principle why a similar type of study could not be made. This paper gives some of the results of such a study using a simple model to represent the turbulent motion. The results given in the present paper are those concerned with the ratios of diffusivities for momentum, energy and mass, and to complete this introduction, a summary is now given of results already obtained in this field by other workers.

1.1. *Existing theories for the prediction of diffusivity ratios*

One of the first proposals for a modification to the Reynolds analogy was made by Jenkins [1] who took into consideration the heat conduction to an eddy during its movement transverse to the main flow. Jenkins initially assumes that the velocity of the eddy remains constant and thus derives an expression for the thermal diffusivity. He later assumes that the Reynolds' analogy applies to the transport process across the entity boundary and is thus able to obtain a relationship for the momentum diffusivity. The result is an expression for the diffusivity ratio, $(\varepsilon_H/\varepsilon_\mu)$, in terms of mixing length, transverse velocity of the eddy and Prandtl number. When evaluated this produces the result shown in Fig. 1. The values predicted by Jenkins for Prandtl numbers of the order of unity are known to be in error by 10–20 per cent. Furthermore the prediction that $(\varepsilon_H/\varepsilon_\mu) \ngtr 1{\cdot}0$ for $P < 1{\cdot}0$ is contrary to experimental evidence. At high Prandtl numbers the Jenkins theory predicts values for $(\varepsilon_H/\varepsilon_\mu)$ which have no upper bound as the Prandtl number increases which again is not found by experiment.

The curve shown in Fig. 2 is that of an expression for $(\varepsilon_H/\varepsilon_\mu)$ derived by Rohsenow and Cohen [2]. In their analysis it was assumed that when an eddy passes through the fluid a temperature gradient is set up in it and that the surface

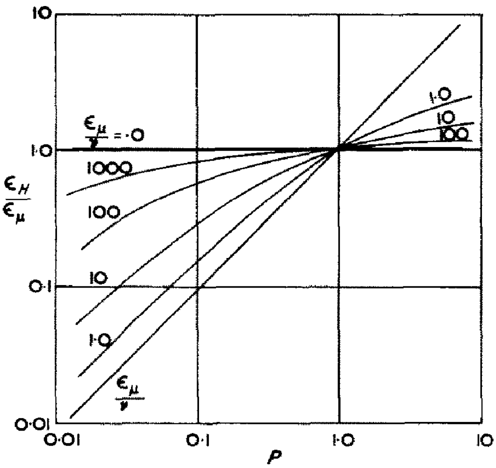

FIG. 1. Relationship between $(\varepsilon_H/\varepsilon_\mu)$ and Prandtl number for various values of ε_μ/ν (Jenkins [1]).

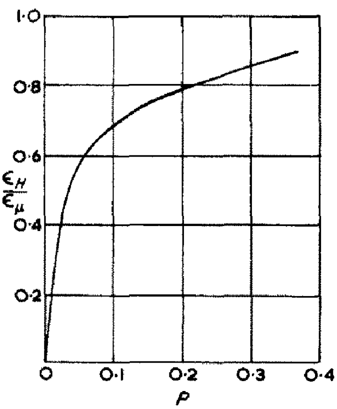

FIG. 2. Relationship between $(\varepsilon_H/\varepsilon_\mu)$ and Prandtl number (Rohsenow and Cohen [2]).

heat-transfer coefficient is infinite. Again the validity of the result can be queried in that it leads to infinite values of $(\varepsilon_H/\varepsilon_\mu)$ as the Prandtl number increases and furthermore no dependence on Reynolds number or turbulence intensity emerges from the analysis, thus contradicting much of the experimental evidence.

A study of the problem has also been made by Deissler [3, 4], using a modified mixing length theory and also by a method based on correlation coefficients. Neither method leads to an expression for the diffusivity ratios which can be compared directly with those given by other authors, but the modified mixing length theory does seem successful in predicting heat transfer in the low Prandtl number range. The theory, however, does contain disposable constants which must be determined by experiment. In Deissler's second paper [4] the correlations between velocities and temperatures at two points in a homogeneous turbulent fluid are derived from the momentum and energy equation. Deissler's results predict that $(\varepsilon_H/\varepsilon_\mu)$ is dependent upon the velocity gradient and that as the gradient increases the value of this ratio approaches unity regardless of the molecular Prandtl number of the fluid. Figure 3 shows the relationship between $(\varepsilon_H/\varepsilon_\mu)$ and Prandtl number for a zero velocity gradient. The results of this work are not directly applicable to non-homogeneous turbulence in a pipe and as a result comparison with experimental results is not of great significance.

A further modification to the mixing length theory has been made by Azer and Chao [5] using an eddy model. In their model it is assumed that a boundary layer round the eddy determines

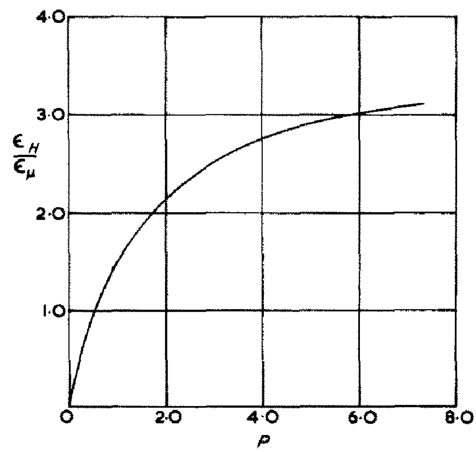

FIG. 3. Variation of $(\varepsilon_H/\varepsilon_\mu)$ with Prandtl number for isotropic turbulence with zero velocity gradient (Deissler [3]).

the heat and momentum transfer to it. The final result is an expression for the diffusivity ratio in terms of various parameters of the turbulence. For a particular type of flow configuration the parameters can be determined by experiment. In the case of pipe flow the predicted diffusivity ratios are shown to be in fair agreement with experimental results.

In the present paper the mixing length concept has been abandoned and a new entity model of a turbulent fluid adopted. Consequently the resulting analysis can not be compared in detail with that of previous theories.

2. A RATIONAL DESCRIPTION OF A TURBULENT FLUID

The approach may be introduced by considering the results of some typical velocity correlation measurements in turbulent flow, shown in Fig. 4. In this figure the correlation R_y

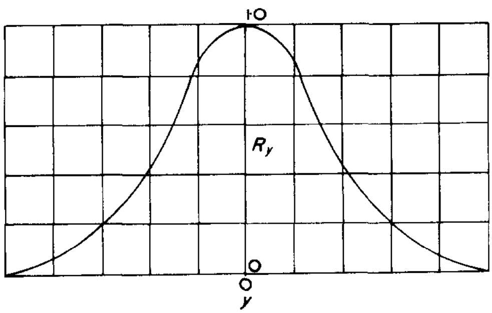

FIG. 4. Measured values of the correlation $R_y = (\bar{u}_0\bar{u}_y/\bar{u}^2)$ in turbulence behind a honeycomb grid.

between the fluctuating components of velocity u_0 and u_y at two points distance y apart in a direction transverse to the mean flow is plotted against the corresponding value for y. The curve shows that R_y decreases to zero within fairly closely defined dimensions which demonstrates that although there is no permanent entity in the fluid there is certainly some type of average zone of influence. This suggests as a limit the possibility that the fluid may be considered as distinct regions each behaving at least temporarily as an entity. A similar type of correlation curve is obtained for other types of turbulent flow and for any direction relative to the flow. Any proposed model for a turbulent fluid must therefore be compatible with this evidence and furthermore since this situation is not found in laminar flow, the model must not produce this type of correlation curve when the flow is laminar.

2.1. *An elementary model of a turbulent fluid*

The notion of dividing a turbulent fluid into regions immediately suggests a model in which the flow field is constructed like a three dimensional jig-saw from rather vague entities of various shapes and sizes as shown in Fig. 5.

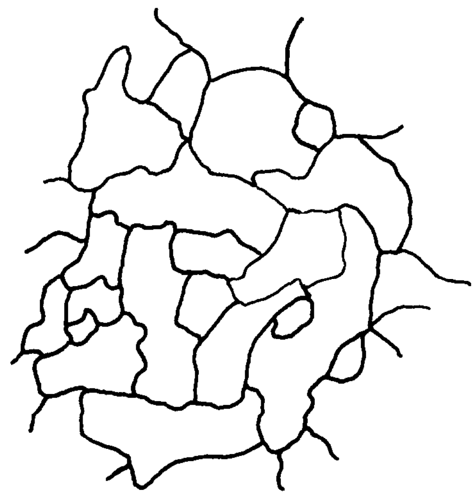

FIG. 5. An ideal entity model of a turbulent fluid.

Within each entity the velocity of the fluid is correlated but there is a rather indistinct boundary across which the mean correlation is zero.

This model, although unmanageable from an analysis point of view, satisfies the required conditions and *is* in principle an exact description of turbulence if the mass, shape, and velocity of an entity is allowed to change with time, i.e. if the entity has not a permanent identity. This ideal model is clearly too complex to be useful but a workable approximation can be made by subdividing into smaller shapes as shown in

Fig. 6. Each of the new entities *a–e* is less distorted from the spherical form than the previous undivided entity.

It is assumed that three variable parameters are necessary for the description of an individual entity, a scale, a shape and a velocity. Interdependence between the parameters for a particular eddy is assumed to be weak, and variation in the average values at different positions in the flow is allowed. It is clear

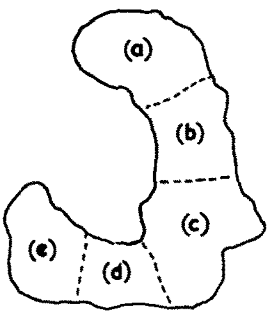

FIG. 6. The subdivision into more regular shapes.

that this generality of description is necessary and that, if sufficient, the bulk behaviour of the fluid can be determined from the collective effect of the motion of the entities. In order to assess the collective effect it is first necessary to analyse the motion of one entity having known parameters and then sum for all allowed values of the parameters. In this way it will be shown that an eddy viscosity and energy conduction coefficient arise naturally from the analysis and can be determined as functions of mean values of the parameters. At a later stage two additional parameters, temperature and concentration, will be introduced enabling thermal diffusion and mass diffusion coefficients to be evaluated, again as functions of the parameters.

The coefficients resulting from this analysis are not in a convenient form for comparison with experimental results, but the ratios of pairs of coefficients can be, and are found to be most consistent with experimentally determined values.

3. THE MOTION OF AN ENTITY ALONG ITS TRAJECTORY

The introduction of the entity shape as a variable presents difficulties when discussing its motion. However, an insight into the way this may be overcome can be had by considering the slow motion of an ellipsoid of semi-principal axes (a, b, c) moving as a solid with velocity $\bar{V} = (U, V, W)$ relative to a fixed coordinate system. Suppose that at some time $t = 0$ it is surrounded by a region of fluid having a general velocity $\bar{v} = (u, v, w)$ then, providing the surrounding fluid is of large volume compared to the ellipsoid, the Reynolds number is small, and the motion is established, it can be shown [6] that the force $\bar{F}$ acting on the ellipsoid is given by

$$\bar{F} = -6\pi\mu r(\bar{V} - \bar{v}) \tag{1}$$

where r is a function of the semi-axes (a, b, c) and when $a = b = c$ then $r = a$.

The equation of motion of the ellipsoid may be written

$$\tfrac{4}{3}\pi r^3 \rho\phi \left(\frac{\mathrm{d}\bar{V}}{\mathrm{d}t}\right) = -6\pi\mu r(\bar{V} - \bar{v})$$

or

$$r^2\phi \left(\frac{\mathrm{d}\bar{V}}{\mathrm{d}t}\right) = -\frac{9\mu}{2\rho}(\bar{V} - \bar{v}) \tag{2}$$

where ϕ is a function of (a, b, c) of value unity when $a = b = c = r$ and may be regarded as a factor which accounts for the effect of the distortion of the entity from the purely spherical shape. Strictly ϕ is also a function of the angular disposition of the major axes to the vector $(\bar{V} - \bar{v})$, however, this dependence will be lost later when a summation is made for the collective behaviour of a group.

Equation (2) can be used to describe the entity motion in a region of fluid having velocity $\bar{v}$ providing suitable values are chosen for r and ϕ. Clearly r and ϕ are not independent and the most rational proposal is to define a new scale parameter R such that the entity volume is $\frac{4}{3}\pi R^3$ and a new distortion factor ψ which retains the value unity whenever the entity is or behaves like a sphere.

Implicit in the use of equation (1) is the assumption that the entity behaves as a solid with no internal velocity gradients. It can be shown [7], however, that for a liquid entity the form of equation (1) remains the same with a change of constant from 6 to 5. For the moment this equation will not be altered for reasons of consistency which will be discussed later.

3.1. *The average motion*

It is inherent in the entity concept that the velocities of adjacent entities are independent. We now assume that the mean behaviour of the entities constitutes the bulk motion of the fluid. Consequently the equation of motion of a particular entity can be written,

$$R^2\psi\left(\frac{d\bar{V}}{dt}\right) = -\frac{9\mu}{2\rho}(\bar{V} - \bar{v}') \qquad (3)$$

where $\bar{v}'$ is the average velocity of the fluid, i.e. the entities, surrounding the entity being considered, and where $\bar{v}'$ is not in any way dependent upon any characteristic of this entity. The *average* motion of the entity for all possible values of $\bar{v}'$ is therefore described by the equation

$$R^2\psi\left(\frac{d\bar{V}}{dt}\right) = -\frac{9\mu}{2\rho}(\bar{V} - \langle\bar{v}'\rangle) \qquad (4)$$

where $\langle\bar{v}'\rangle$ is the expected velocity of the surrounding fluid, or in other words $\langle\bar{v}'\rangle$ is the bulk fluid velocity.

The dissipation relationship is formed by taking the scalar product of equation (4) with the entity velocity

$$R^2\psi\frac{d(\bar{V}^2/2)}{dt} = -\frac{9\mu}{2\rho}(\bar{V} - \langle\bar{v}'\rangle).\bar{V}. \qquad (5)$$

Taking the expected value of both sides of this equation for all possible values of R, ψ, $\bar{V}$ then

$$\left\langle R^2\psi\frac{d(\bar{V}^2/2)}{dt}\right\rangle = -\frac{9\mu}{2\rho}\langle\bar{V}^2 - \langle\bar{v}'\rangle.\bar{V}\rangle \qquad (6)$$

or if the correlations between R, ψ, and $\bar{V}$ are weak then this can be written

$$\langle R^2\rangle\langle\psi\rangle\left\langle\frac{d(\bar{V}^2/2)}{dt}\right\rangle = -\frac{9\mu}{2\rho}(\langle\bar{V}^2\rangle - \langle\bar{v}'\rangle^2) \qquad (7)$$

since $\langle\bar{V}\rangle$ = average entity velocity = average fluid velocity = $\langle\bar{v}'\rangle$.

This result can be simplified somewhat by considering the case of steady flow and referring the entity velocity to the mean fluid velocity $\langle\bar{v}'\rangle$.

This results in a version of the well known Von Kármán–Howarth equation for the rate of energy dissipation and has the form

$$\left\langle\frac{d(\bar{V}^2/2)}{dt}\right\rangle = -\frac{9\mu\langle\bar{V}_r^2\rangle}{2\rho\langle R^2\rangle\langle\psi\rangle}. \qquad (8)$$

For isotropic turbulence this dissipation equation is usually given in the form

$$\left\langle\frac{d(\bar{V}^2/2)}{dt}\right\rangle = -\frac{G\mu\langle\bar{V}_r^2\rangle}{\rho\delta^2} \qquad (9)$$

where G is a numerical constant and δ is defined as the microscale of the turbulence.

For isotropic turbulence $\langle\psi\rangle$ is independent of direction and a comparison of equations (8) and (9) shows that the product $\langle R^2\rangle\langle\psi\rangle$ is proportional to the square of the microscale. Equations (8) and (9) provide a link between this treatment and one based on correlation coefficients. The microscale δ corresponds to a mean entity size. In correlation treatments it is said to represent a mean eddy size. Hence there is a correspondence between the "entities" and the usual "eddies".

The entity concept leads thus to an equation of the same form as that obtained using the more conventional statistical analysis of correlation coefficients, and has corresponding parameters. It has been pointed out by Silver [8] that it should be possible to discuss the problem of turbulence in terms of a statistical treatment of the production, diffusion and decay of a collection of entities. Clearly now the results of such a treatment can if necessary be related to an approach based on correlations. There is, however, much to be said in favour of having an entity model to discuss particularly when there are energy and mass transfers as well as momentum transfer.

4. MOMENTUM TRANSPORT IN A TURBULENT FLUID

In the simple case where the bulk flow is unidirectional with constant velocity gradient, the bulk velocity relative to some plane $y = 0$ can be represented by the vector:

$$\langle \bar{v}' \rangle = \left(y \frac{d\langle u \rangle}{dy}, 0, 0 \right)$$

where $d\langle u \rangle/dy$ is the gradient of the bulk velocity.

It follows from equation (4) that the average behaviour of entities having the parameters R, ψ satisfy the equations

$$\left.\begin{aligned} R^2\psi \frac{dU}{dt} &= -\frac{9\mu}{2\rho}\left(U - y\frac{d\langle u \rangle}{dy}\right) \\ R^2\psi \frac{dV}{dt} &= -\frac{9\mu V}{2\rho} \\ R^2\psi \frac{dW}{dt} &= -\frac{9\mu W}{2\rho}. \end{aligned}\right\} \quad (10)$$

Now if the initial conditions are $(U, V, W) = (U_0, V_0, W_0)$ when $y = 0$ and the bulk flow velocity is steady, then the solutions to equations (10) are

$$\left.\begin{aligned} U &= U_0\left(1 - \frac{y}{\lambda^*}\right) \\ &\quad + \frac{d\langle u \rangle}{dy}\left[y + \lambda^*\left(1 - \frac{y}{\lambda^*}\right)\ln\left(1 - \frac{y}{\lambda^*}\right)\right] \\ V &= V_0\left(1 - \frac{y}{\lambda^*}\right) \\ W &= W_0\left(1 - \frac{y}{\lambda^*}\right) \end{aligned}\right\} \quad (11)$$

where $\lambda^* = (2\rho R^2 V_0 \psi/9\mu)$ is the mean distance travelled by the entity from the plane $y = 0$ before its momentum is entirely dissipated by viscous action. Since the decay of entity momentum is exponential, an infinite time is required for the entity to travel the distance λ^*. It thus represents a limit which will be rapidly approached but never achieved. It will be assumed in the subsequent analysis that the difference between λ^* and any actual limit is negligible. The length parameter λ^* may also be written in terms of the initial Reynolds number of the entity as

$$\lambda^* = \frac{2R\psi N_R}{9}. \quad (12)$$

A comparison between equations (11) shows that the axial component of the entity momentum is influenced by the mean velocity gradient and thus gives rise to a shear stress which can now be determined.

4.1. *Shear stress due to momentum flux*

Since an entity is identified by its motion relative to the mean flow it follows for consistency that entities are also created within the flow field and moreover created by some rapid process. This instant creation is necessary since equations (11) show that any prolonged motion dissipates momentum and hence identity.

Consider now the flux of entities crossing the plane $y = 0$ and in particular an entity that was created a distance λ from the plane and is now crossing it. If ΔU and ΔV are the components of the entity velocity relative to the average fluid velocity in the plane $y = 0$ then the contribution to the kinetic shear stress from this entity is given by

$$\sigma = -\rho\,\Delta U\,\Delta V. \quad (13)$$

But from equations (11)

$$\Delta U = U_0\left(1 - \frac{\lambda}{\lambda^*}\right) + \frac{d\langle u \rangle}{dy}\lambda^*\left(1 - \frac{\lambda}{\lambda^*}\right) \times \ln\left(1 - \frac{\lambda}{\lambda^*}\right) \quad (14)$$

and

$$\Delta V = V_0\left(1 - \frac{\lambda}{\lambda^*}\right). \quad (15)$$

Here U_0 and V_0 are the values of U and V at creation of the entity relative to the mean velocity in that region.

Hence by substitution in equation (13)

$$\sigma = -\rho U_0 V_0 \left(1 - \frac{\lambda}{\lambda^*}\right)^2 - \rho \frac{\mathrm{d}\langle u \rangle}{\mathrm{d}y} V_0 \lambda^* \left(1 - \frac{\lambda}{\lambda^*}\right)^2 \ln \left(1 - \frac{\lambda}{\lambda^*}\right). \quad (16)$$

The average or expected shear stress τ may now be found by taking the expected value of the right-hand side of equation (16) for all entities which pass through the plane $y = 0$. Thus

$$\tau = \langle \sigma \rangle = -\left\langle \rho U_0 V_0 \left(1 - \frac{\lambda}{\lambda^*}\right)^2 \right\rangle - \rho \frac{\mathrm{d}\langle u \rangle}{\mathrm{d}y} \left\langle V_0 \lambda^* \left(1 - \frac{\lambda}{\lambda^*}\right)^2 \ln \left(1 - \frac{\lambda}{\lambda^*}\right) \right\rangle \quad (17)$$

but $\langle \rho U_0 V_0 (1 - \lambda/\lambda^*)^2 \rangle = 0$ providing the creation process favours neither positive nor negative values of U_0. Hence

$$\tau = -\rho \frac{\mathrm{d}\langle u \rangle}{\mathrm{d}y} \left\langle V_0 \lambda^* \left(1 - \frac{\lambda}{\lambda^*}\right)^2 \ln \left(1 - \frac{\lambda}{\lambda^*}\right) \right\rangle \quad (18)$$

or using equation (12)

$$\tau = -\frac{2\mu}{9} \left\langle \psi N_R^2 \left(1 - \frac{\lambda}{\lambda^*}\right)^2 \times \ln \left(1 - \frac{\lambda}{\lambda^*}\right) \right\rangle \frac{\mathrm{d}\langle u \rangle}{\mathrm{d}y}. \quad (19)$$

It can be seen from this equation that an "eddy viscosity" is generated naturally by analysis of the entity model, furthermore its value is clearly dependent upon the previous history of the entities passing through a region of fluid and not on purely local parameters as in a perfect gas. The terms within the expectation sign of equation (19) can be separated into two distinct parts (ψN_R^2) and $(1 - \lambda/\lambda^*)^2 \ln (1 - \lambda/\lambda^*)$, the former being concerned with the initial condition of the entity, the latter being a function of (λ/λ^*), i.e. the proportion of the maximum path that has been traversed. Consequently,

$$\left\langle \psi N_R^2 \left(1 - \frac{\lambda}{\lambda^*}\right)^2 \ln \left(1 - \frac{\lambda}{\lambda^*}\right) \right\rangle = \langle \psi N_R^2 \rangle \left\langle \left(1 - \frac{\lambda}{\lambda^*}\right)^2 \ln \left(1 - \frac{\lambda}{\lambda^*}\right) \right\rangle. \quad (20)$$

However, given the information that there is continuous production and dissipation of entities and that there is no mean flow in the transverse direction, then the least prejudiced assignment of probability density to the value of (λ/λ^*) when an entity crosses the plane $y = 0$ is that the density is uniform for all values of (λ/λ^*) between 0 and 1. Thus all values of (λ/λ^*) between 0 and 1 are equally likely and hence

$$\left\langle \left(1 - \frac{\lambda}{\lambda^*}\right)^2 \ln \left(1 - \frac{\lambda}{\lambda^*}\right) \right\rangle$$

$$\int_0^1 \left(1 - \frac{\lambda}{\lambda^*}\right)^2 \ln \left(1 - \frac{\lambda}{\lambda^*}\right) \mathrm{d}\left(\frac{\lambda}{\lambda^*}\right) = -\frac{1}{9}. \quad (21)$$

Thus using equations (19–21)

$$\tau = \frac{2\mu}{81} \langle \psi N_R^2 \rangle \frac{\mathrm{d}\langle u \rangle}{\mathrm{d}y} = \varepsilon_\mu \frac{\mathrm{d}\langle u \rangle}{\mathrm{d}y}. \quad (22)$$

It is seen from equation (22) and the preceding analysis that the concept of an eddy viscosity ε_μ arises naturally from the entity model of a turbulent fluid. It is also clear that its value in any region of fluid is dependent on the history of the entities traversing the region and in this respect cannot be described as a local parameter. By analogy one would expect that it should now be possible with a similar analysis to investigate the diffusion of energy, thermal and turbulent, in the fluid and arrive at expressions for the appropriate transport coefficients. This will now be done and the results will be shown to be similar to those already obtained in the case of momentum transport. Because of this similarity it will be possible to predict numerical values for the ratios of the various transport coefficients and compare these with experimental results.

5. ENERGY AND MASS TRANSPORT IN A TURBULENT FLUID

5.1. *Turbulent energy*

The simplified case is again considered where there is a uniform distribution of the average turbulent energy in any plane perpendicular to the "Y" axis. It is assumed where necessary that the effect of any mean velocity gradient on the energy diffusion is negligible though the effect of this could be accounted for if necessary. As in the previous analysis an entity is considered to have a velocity (U, V, W) when it has traversed a distance "y" in a direction parallel to the "Y" axis since its creation when its velocity was (U_0, V_0, W_0). Thus from equation (11) the relationships between the initial and subsequent velocities are

$$\left.\begin{aligned} U &= U_0\left(1 - \frac{y}{\lambda^*}\right) \\ V &= V_0\left(1 - \frac{y}{\lambda^*}\right) \\ W &= W_0\left(1 - \frac{y}{\lambda^*}\right). \end{aligned}\right\} \quad (23)$$

Hence

$$\tfrac{1}{2}(U^2 + V^2 + W^2) = \tfrac{1}{2}(U_0^2 + V_0^2 + W_0^2) \times (1 - y/\lambda^*). \quad (24)$$

Thus if an entity is created and subsequently moves a distance λ in the "y" direction in a fluid having a constant energy gradient $d\langle E\rangle/dy$ then the average energy density difference between the entity and its new surroundings ΔE is given by

$$\Delta E = \left(1 - \frac{\lambda}{\lambda^*}\right)^2 \lambda \frac{d\langle E\rangle}{dy}. \quad (25)$$

Thus the energy flux q due to this entity as it crosses a plane perpendicular to the "Y" axis is

$$q = \rho V\left(1 - \frac{\lambda}{\lambda^*}\right)^2 \lambda \frac{d\langle E\rangle}{dy} \quad (26)$$

or using equations (12) and (23)

$$q = \frac{2\mu}{9}\psi N_R^2 \left(\frac{\lambda}{\lambda^*}\right)\left(1 - \frac{\lambda}{\lambda^*}\right)^3 \frac{d\langle E\rangle}{dy}. \quad (27)$$

Thus the expected flux $Q = \langle q\rangle$ can be found by taking the expected value of the terms on the right of equation (27). Thus

$$Q = \frac{2\mu}{9}\langle\psi N_R^2\rangle \left\langle\left(\frac{\lambda}{\lambda^*}\right)\left(1 - \frac{\lambda}{\lambda^*}\right)^3\right\rangle \frac{d\langle E\rangle}{dy}$$

$$= \frac{2\mu}{9}\langle\psi N_R^2\rangle \left[\int_0^1 \left(\frac{\lambda}{\lambda^*}\right)\left(1 - \frac{\lambda}{\lambda^*}\right)^3 d\left(\frac{\lambda}{\lambda^*}\right)\right] \times \frac{d\langle E\rangle}{dy}.$$

Therefore

$$Q = \frac{\mu}{90}\langle\psi N_R^2\rangle \frac{d\langle E\rangle}{dy}. \quad (28)$$

Defining the coefficient of diffusivity of turbulent energy by the relationship

$$Q = \varepsilon_e \frac{d\langle E\rangle}{dy} \quad (29)$$

it follows that

$$\varepsilon_e = \frac{\mu}{90}\langle\psi N_R^2\rangle. \quad (30)$$

Again a transport coefficient is generated by the analysis and the coefficient is again proportional to the product $\mu\langle\psi N_R^2\rangle$. By comparing equations (30) and (22) it can be seen that the ratio $(\varepsilon_e/\varepsilon_\mu)$ is predicted to be 0·45 which is compared in Table 1 with values from a recent paper by Spalding [9]. In this paper Spalding quotes the experimental results obtained by Wieghardt [10] and Glushko [11] and compares these with his own relationships for ε_μ and ε_e. The ratios of these two quantities are shown to be pure numbers and can be compared with the present prediction. The relationships given by

Table 1

Author	$(\varepsilon_e/\varepsilon_u)$
Wieghardt	0·679*
Glushko	0·4*
Spalding	0·587
Present	0·45

* Experimental result.

Spalding for ε_μ and ε_e are not in a form suitable for comparison with equations (22) and (30).

It is clear from a comparison of these results that the true value of $(\varepsilon_e/\varepsilon_\mu)$ is a little obscure; however, the entity model predicts a value of 0·45 which lies between the two experimental results. It is worth noting at this point that this result is independent of whether the constant 5 or 6 is used in Stokes' Law and thus is independent of whether the entity is treated as solid or liquid.

5.2. *Thermal energy*

The process of thermal energy diffusion is carried out by the migration of entities between regions of differing energy and in many respects the analysis of this and the process of turbulent energy diffusion are similar. In the latter case it was shown that Stokes' Law can be used to determine the dissipation of energy as the entity passes through the fluid and what is now required is an analysis to determine the manner in which the thermal energy of the entity is changed in passage through the fluid. In order to ensure that the reasoning here is consistent it is useful to consider the implications of using Stokes' Law as if the entity were a solid body. Since it is known in fact to be fluid the assumption of solid behaviour implicitly assumes instant propagation of uniform motion through it. The actual absence of such uniformity causes the numerical constant in Stokes' Law to be reduced in value. It is a moot point whether either extreme prevails in practice or whether the motion is something between the two. The important point is that for consistency an assumed uniformity of momentum within the entity will also necessarily mean an assumption of a uniformity of temperature. Thus consistency requires that, if in momentum interactions, the entity is treated as solid, in thermal interaction it must be assumed to have a uniform internal temperature.

With the preceding argument in mind the thermal interaction of an entity of uniform temperature T surrounded by a fluid of temperature T_f is now considered. If the entity is spherical then the heat transfer across its surface $\dot{q}$ is given by

$$\dot{q} = 4\pi a K(T - T_f) \tag{31}$$

where K is the thermal conductivity of the fluid, and a the sphere radius.

As in the case of momentum transfer the interaction process is assumed to be quasi-static so that equation (31) holds even when the temperature of the surrounding fluid is changing. Hence

$$\dot{q} = -\tfrac{4}{3}\pi a^3 \rho c_p \left(\frac{\mathrm{d}T}{\mathrm{d}t}\right) = 4\pi a K(T - T_f) \tag{32}$$

or

$$\left(\frac{\mathrm{d}T}{\mathrm{d}t}\right) = -\frac{3K}{\rho c_p a^2}(T - T_f). \tag{33}$$

Without loss of generality a distortion factor ψ' can now be introduced into equation (33) to account for distortion of the entity from the spherical shape, with the result that for an entity of general shape

$$\left(\frac{\mathrm{d}T}{\mathrm{d}t}\right) = -\frac{3K}{\rho c_p R^2 \psi'}(T - T_f). \tag{34}$$

The corresponding result for a momentum interaction being

$$\left(\frac{\mathrm{d}V}{\mathrm{d}t}\right) = -\frac{9\mu}{2\rho R^2 \psi}(V - V_f). \tag{35}$$

Equation (34) for the thermal interaction may now be used in exactly the same manner as equation (35) in order to find the average behaviour of the entity. Clearly in this case the fluid temperature T_f is a random variable and hence if its average or expected value is $\langle T_f \rangle$ then the average behaviour of the entity is described by the equation

$$\left(\frac{\mathrm{d}T}{\mathrm{d}t}\right) = -\frac{3K}{\rho c_p R^2 \psi'}(T - \langle T_f \rangle). \tag{36}$$

In the simplified case where the temperature in planes perpendicular to the "Y" axis is uniform and the temperature gradient is constant, then $\langle T_f \rangle$ can be expressed in terms of the temperature gradient $\mathrm{d}\langle T_f \rangle/\mathrm{d}y$, and hence, upon substituting in equation (36) and referring all temperatures to that in the plane $y = 0$, it is found that

$$\left(\frac{\mathrm{d}T}{\mathrm{d}t}\right) = -\frac{3K}{\rho c_p R^2 \psi'}\left(T - y\frac{\mathrm{d}\langle T_f \rangle}{\mathrm{d}y}\right). \quad (37)$$

Using the solutions to equation (35) as given in equation (23), the above equation can now be solved giving

$$T = T_0\left(1 - \frac{y}{\lambda^*}\right)^{\alpha/\beta} + y\frac{\mathrm{d}\langle T_f\rangle}{\mathrm{d}y} - \frac{V_0}{(\beta - \alpha)} \times \left[\left(1 - \frac{y}{\lambda^*}\right)^{\alpha/\beta} - \left(1 - \frac{y}{\lambda^*}\right)\right]\frac{\mathrm{d}\langle T_f\rangle}{\mathrm{d}y} \quad (38)$$

where $\alpha = (3K/\rho c_p R^2 \psi')$, $\beta = (9\mu/2\rho R^2 \psi)$ and T_0 is the entity temperature at creation.

Thus the average temperature difference between the entity and the surrounding fluid as it crosses the plane $y = 0$ is given by

$$\Delta T = T_0\left(1 - \frac{\lambda}{\lambda^*}\right)^{\alpha/\beta} - \frac{V_0}{(\beta - \alpha)}\left[\left(1 - \frac{\lambda}{\lambda^*}\right)^{\alpha/\beta} - \left(1 - \frac{\lambda}{\lambda^*}\right)\right]\frac{\mathrm{d}\langle T_f\rangle}{\mathrm{d}y} \quad (39)$$

and hence the energy flux q_H is

$$q_H = \rho c_p \Delta V \Delta T = \rho c_p T_0 V_0\left(1 - \frac{\lambda}{\lambda^*}\right)^{(\alpha+\beta)/\beta} - \frac{\rho c_p V_0^2}{(\beta - \alpha)} \times \left[\left(1 - \frac{\lambda}{\lambda^*}\right)^{(\alpha+\beta)/\beta} - \left(1 - \frac{\lambda}{\lambda^*}\right)^2\right]\frac{\mathrm{d}\langle T_f\rangle}{\mathrm{d}y}. \quad (40)$$

The expected energy flux may now be found from equation (40), as has been done previously for momentum flux, and results in the following expression for the energy flux $Q_H = \langle q_H \rangle$

$$Q_H = \frac{\mu}{9}\langle \psi' N_R^2 \rangle\left(\frac{P}{1 + 3P(\psi'/\psi)}\right)\frac{\mathrm{d}\langle T_f\rangle}{\mathrm{d}y} \quad (41)$$

where $P = \mu c_p/K$ = molecular Prandtl number number of the fluid. Defining the diffusivity of thermal energy ε_H by the relationship

$$Q_H = \varepsilon_H \frac{\mathrm{d}\langle T_f\rangle}{\mathrm{d}y} \quad (42)$$

then

$$\varepsilon_H = \frac{\mu}{9}\langle \psi' N_R^2 \rangle\left(\frac{P}{1 + 3P(\psi'/\psi)}\right) \quad (43)$$

Again a diffusion coefficient has been generated by the analysis and its dependence on the history of the entities can be seen in equation (43). In this analysis, however, it was necessary to introduce the distortion factor ψ' and in order to allow further comparison between the various diffusivities it is necessary to find the relationship between ψ' and the previous distortion factor ψ.

It is clear from the definition of ψ and ψ' that for an undistorted spherical entity each takes the value unity and hence $(\psi/\psi') = 1{\cdot}0$. However, an extreme example of distortion is when the entity has a disc shape, in which case the ratio (ψ/ψ') can be shown to lie between 0·74 and 1·1, the actual value depending upon the direction of motion relative to the axis of the disc. Consequently for small distortion from the spherical shape little error will be incurred by assuming $(\psi/\psi') = 1{\cdot}0$. In close proximity to a boundary wall where the entity distortion may be severe this may not be valid.

With the ratio (ψ/ψ') established it is now possible to compare the ratio $(\varepsilon_H/\varepsilon_\mu)$ with experimental evidence, but before doing so it is worthwhile investigating mass transport in a turbulent fluid.

5.3. *Mass*

The process of mass transfer by molecular motion can be shown to be described by the same fundamental equation as heat transfer by conduction in a homogeneous substance. The only distinction is that temperature gradients in the latter are replaced by concentration gradients in the former and also the thermal diffusivity is replaced by a mass diffusivity, both being properties of the substance. As a result the mass transfer to a spherical entity can be described by equation (31) with the appropriate changes in nomenclature, i.e.

$$\dot{m} = 4\pi a D(C - C_f) \tag{44}$$

where $\dot{m}$ is the rate of mass transfer across the spherical surface of one component and C, C_f are the corresponding concentrations of this component in the sphere and surrounding fluid respectively. All the arguments and analysis subsequently made for the case of thermal energy transport can now be made again simply by an adjustment of symbols and hence the result is the generation of a diffusion coefficient for mass transport in the form

$$\varepsilon_m = \frac{\mu}{9}\langle \psi' N_R^2 \rangle \left(\frac{S}{1 + 3S\,(\psi'/\psi)}\right). \tag{45}$$

This equation corresponds with equation (43) with the replacement of the molecular Prandtl number $P = (\mu c_p/K)$ by the Schmidt number $S = (\mu/D)$ Furthermore the subsequent proof that $(\psi/\psi') = 1$ is applicable and hence equation (45) may now be written

$$\varepsilon_m = \frac{\mu}{9}\langle \psi N_R^2 \rangle \left(\frac{S}{1 + 3S}\right). \tag{46}$$

Using equations (22, 43, 46) it is now convenient to find the ratios of the various diffusivities which may then be compared with experimental evidence. The similarity between the expressions for the thermal and mass diffusion coefficients will also allow the results of experimentation in mass transfer to supplement that in heat transfer and vice-versa.

5.4. *A comparison with experimental evidence in heat and mass transfer*

The diffusivity ratios which are of interest in the field of heat and mass transfer are $(\varepsilon_H/\varepsilon_\mu)$ and $(\varepsilon_m/\varepsilon_\mu)$ these ratios being the inverse of the turbulent Prandtl and Schmidt numbers respectively. Using the relationships of equations (22, 43, 46), these may now be expressed as follows

$$\frac{\varepsilon_H}{\varepsilon_\mu} = \frac{1}{P_T} = \gamma\left(\frac{9P}{2 + 6P}\right) \tag{47}$$

and

$$\frac{\varepsilon_m}{\varepsilon_\mu} = \frac{1}{S_T} = \gamma\left(\frac{9S}{2 + 6S}\right) \tag{48}$$

where γ is a factor of approximately unity to allow for the approximations made in the derivation Sf equations (47) and (48). The constant takes into account slight differences between ψ and ψ' and the fact that the assumptions of constant entity temperature and velocity may not cancel out.

In both cases the expression on the right has the same general form and is independent of any characteristic of the flow such as Reynolds number. A comparison with experimental work can now be made since the values of $(\varepsilon_H/\varepsilon_\mu)$ and $(\varepsilon_m/\varepsilon_\mu)$ have been found on numerous occasions for different fluids. In some instances the experimental evidence shows that $(\varepsilon_H/\varepsilon_\mu)$ and $(\varepsilon_m/\varepsilon_\mu)$ are not completely independent of the Reynolds number and hence for a first comparison, Fig. 7 compares the results of equations (47) and (48) with results obtained or extrapolated to a moderate Reynolds number of the order of 2×10^4. The value of γ is taken to be one.

It is clear from the comparison made in Fig. 7 that the predicted curve shows the right trend and that the values given for various fluids are of the right order. This evidence, when combined with that given previously for the case of turbulent energy diffusion, fully justifies the model for the turbulent fluid originally proposed.

6. THE EFFECT OF INCREASING TURBULENCE INTENSITY

In the preceding analysis it was assumed that transport between entities was due solely to molecular transport and that the effect of

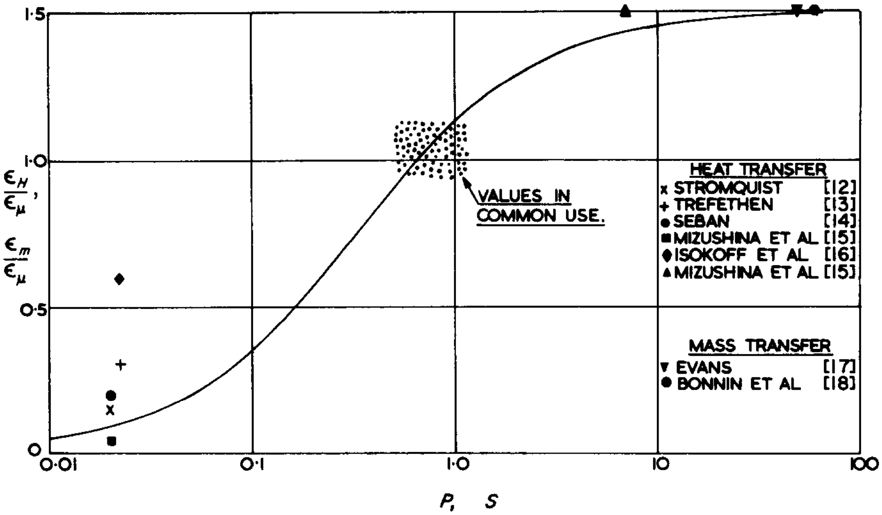

FIG. 7. Predicted and experimental results for the effect of Prandtl and Schmidt numbers on the corresponding ratios of turbulent diffusivities.

smaller entities penetrating the boundary between large entities was unimportant. It is clear from equations (22, 30, 43, 46) that entities of large Reynolds numbers play the dominant role in determining the transport coefficients, but it is also known [19] that it is the small entities that are responsible for the eventual dissipation of turbulent energy into heat. Thus at higher flow Reynolds number where the energy dissipation is greater, the smaller entities are more abundant. Consequently the transport process across the boundary of a large entity is likely to be influenced by the migration of small scale entities across the surface. The degree of magnitude of this effect will clearly depend upon the magnitude of the energy dissipation and hence on the Reynolds number of the flow through a given system. It will be shown how the values of $(\varepsilon_H/\varepsilon_\mu)$ and $(\varepsilon_m/\varepsilon_\mu)$ may be found under conditions where the transport process between large entities is dominanted by the small entity migration.

Consider the situation shown in Fig. 8 where transport between a large entity and its neighbours is accomplished by the migration of small scale entities across the boundary. Under these conditions the transport between the large entities is not determined by the molecular transport coefficients of the fluid, but by the transport coefficients appropriate to the small scale entity system, i.e. by ε_μ, ε_H and ε_m. In all other respects the process is, however, exactly the same as that considered in the previous analysis for molecular interaction between the large scale entities. It is therefore correct to describe the process using exactly the same

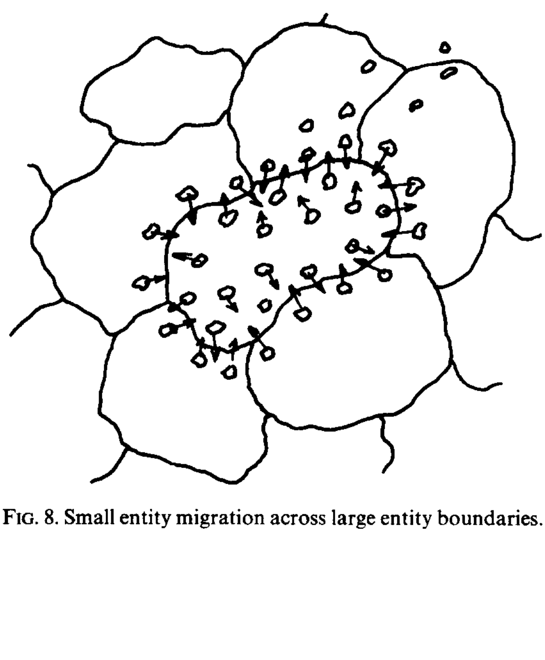

FIG. 8. Small entity migration across large entity boundaries.

basic relationships as in the previous analysis except that the molecular coefficients μ, $K/\rho c_p$, D must be replaced by the appropriate transport coefficients for the small scale entity system ε_μ, ε_H, and ε_m. Without repeating the analysis it is clear that this will result in the formation of new transport coefficients for the large entity system ε_μ^+, ε_H^+, ε_m^+ given by

$$\varepsilon_\mu^+ = \frac{2\varepsilon_\mu}{81}\langle\psi^+ N_R^{+2}\rangle = \left(\frac{2}{81}\right)^2 \mu\langle\psi N_R^2\rangle\langle\psi^+ N_R^{+2}\rangle \quad (49)$$

$$\varepsilon_H^+ = \frac{\varepsilon_\mu}{9}\langle\psi^+ N_R^{+2}\rangle\left(\frac{P_T}{1+3P_T}\right) = \frac{2\mu}{9^3}\langle\psi N_R^2\rangle\langle\psi^+ N_R^{+2}\rangle\left(\frac{P_T}{1+3P_T}\right) \quad (50)$$

$$\varepsilon_m^+ = \frac{\varepsilon_\mu}{9}\langle\psi^+ N_R^{+2}\rangle\left(\frac{S_T}{1+3S_T}\right) = \frac{2\mu}{9^3}\langle\psi N_R^2\rangle\langle\psi^+ N_R^{+2}\rangle\left(\frac{S_T}{1+3S_T}\right) \quad (51)$$

where ψ^+ and N_R^+ are the scale and Reynolds number parameters for the large scale entity system. The ratios $(\varepsilon_H^+/\varepsilon_\mu^+)$, $(\varepsilon_m^+/\varepsilon_\mu^+)$ are found to be given by

$$\frac{\varepsilon_H^+}{\varepsilon_\mu^+} = \gamma^+\left(\frac{9P_T}{2+6P_T}\right) = \frac{\gamma^+(3+9P)}{\gamma(2+6P+3\gamma P)} \doteqdot \frac{(3+9P)}{(2+9P)} \quad (52)$$

$$\frac{\varepsilon_m^+}{\varepsilon_\mu^+} = \gamma^+\left(\frac{9S_T}{2+6S_T}\right) = \frac{\gamma^+(3+9S)}{\gamma(2+6S+3\gamma S)} \doteqdot \frac{(3+9S)}{(2+9S)}. \quad (53)$$

The other ratio $(\varepsilon_e/\varepsilon_\mu)$ which was found to have the value 0·45 retains this value since it is independent of any transport properties of the entity interaction. In Fig. 9 the values of $(\varepsilon_H^+/\varepsilon_\mu^+)$ and $(\varepsilon_m^+/\varepsilon_\mu^+)$ are shown plotted together with the relationship for $(\varepsilon_H/\varepsilon_\mu)$ and $(\varepsilon_m/\varepsilon_\mu)$. Also shown in Fig. 9 are the experimental points at low Reynolds number flows, previously given in Fig. 7, together with corresponding results for high Reynolds numbers. The predicted shift in the diffusivity ratios are obviously in the right direction and furthermore the most extreme experimental values are in fact very close to the predicted values. For Prandtl and Schmidt numbers of the order of 0·7/1·0 there is no experimental evidence of any consistent Reynolds number effect. In Fig. 6 it is clear that within this range the effect is only of the order of 10 per cent which is of the order of the error in this type of experiment. The experimental results are therefore completely consistent with the predictions.

Clearly the step from equations (47) and (48) to (52) and (53) can be repeated many times to obtain the diffusivity ratios at higher orders of turbulence intensity. Ultimately the ratios $(\varepsilon_H/\varepsilon_\mu)$ and $(\varepsilon_m/\varepsilon_\mu)$ converge to a limit of approximately 1·1. Figure 10 shows in diagramatic form the variation in diffusivity ratio with increasing turbulence intensity for a fluid of low and high molecular Prandtl number. No experimental evidence has been found to confirm the oscillation about the limit as shown in these curves. It may be that the scale or Reynolds numbers of experimental flows have been too small to demonstrate this effect.

7. CONCLUSIONS

From the present investigation it is concluded that the entity model of a turbulent fluid is a useful concept for predicting the bulk behaviour. The values predicted for the diffusivity ratios are in agreement with experimental results and have been obtained without the use of any adjustable constants. Furthermore a framework has been generated within which the turbulent fluid may be described more simply than by the use of mixing length or correlation concepts. The comparative simplicity of the model and the subsequent analysis makes application to other problems in turbulence tractable and it is hoped will promote further work. In particular the present authors have under investigation the

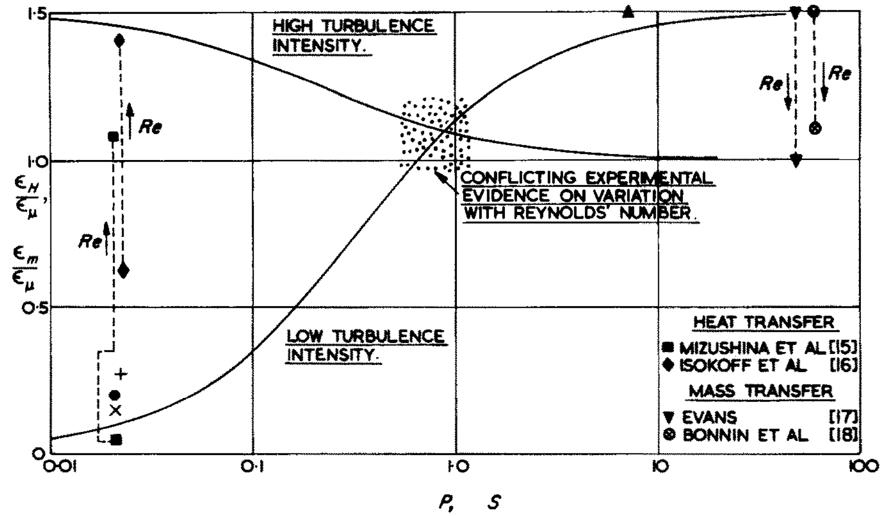

FIG. 9. Predicted and experimental results for the effect of turbulence intensity on the ratios of turbulent diffusivities.

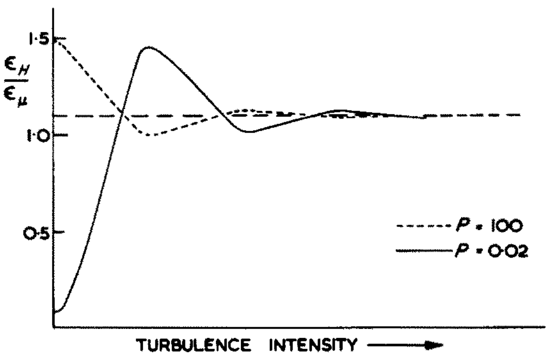

FIG. 10. Variation in diffusivity ratio with increasing turbulence intensity.

problem of predicting the transition to turbulent flow and that of the turbulent flow in a pipe.

In short, while the transport processes occurring in a turbulent fluid are in actuality no doubt more complex than this simple model, it does appear to have utility for application which can be lacking in a more complex analysis.

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Résumé—Les propriétés de transport d'un fluide turbulent sont étudiées en employant une méthode simple pour représenter le comportement détaillé du fluide. Dans le modèle, ceci est attribué aux mouvements de particules fluides de taille, de forme et de vitesse variable, et une analyse est faite pour trouver l'effet sur le système entier, d'interactions et du transport entre les particules individuelles. L'analyse permet de trouver des expressions pour les diffusivités de masse, de quantité de mouvement et d'énergie en fonction des propriétés de la turbulence. Des valeurs expérimentales des différents rapports de diffusivité sont en accord favorable avec les prévisions théoriques. On en conclut que le modèle est un concept utile pour prédire le comportement global dans l'écoulement turbulent.

Zusammenfassung—Die Transporteigenschaften einer turbulenten Flüssigkeit werden mit Hilfe einer einfachen Methode untersucht, welche das Flüssigkeitsverhalten im einzelnen wiedergibt. Im Modell wird dies den Bewegungen der Flüssigkeitsarten von unterschiedlicher Grösse, Form und Geschwindigkeit zugeordnet und es wird eine Analyse durchgeführt, um den Einfluss von Wechselwirkungen und Transport zwischen den einzelnen Arten auf das ganze System zu finden. Die Analyse liefert Ausdrücke für die Massen-, Impuls- und Energieausbreitung in Abhängigkeit von den Turbulenzeigenschaften.

Experimentelle Werte der verschiedenen Ausbreitungsverhältnisse stimmen gut mit theoretischen Berechnungen überein. Das Modell wird als nützlich zur Berechnung des Hauptstromverhaltens in turbulenter Strömung angesehen.